

$\pi\pi$ phase shifts from $K \rightarrow 2\pi$

The FlaviaNet Kaon Working Group ^{*†}

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Abstract

We update the numerical results for the s-wave $\pi\pi$ scattering phase-shift difference $\delta_0^0 - \delta_0^2$ at $s = m_K^2$ from a previous study of isospin breaking in $K \rightarrow 2\pi$ amplitudes in chiral perturbation theory. We include recent data for the $K_S \rightarrow \pi\pi$ and $K^+ \rightarrow \pi^+\pi^0$ decay widths and include experimental correlations.

The authors of Refs. 1 and 2 have shown the importance of taking into account radiative corrections in $K \rightarrow \pi\pi$ decays both in experimental measurements and theoretical predictions. In particular, these corrections are enhanced in the extraction of the phase shifts $\delta_0^0 - \delta_0^2$ by the $\Delta I = 1/2$ rule by a factor of ~ 22 . On the experimental side, KLOE has measured with high accuracy the ratio of branching ratios (BRs) for the decay $K_S \rightarrow \pi^+\pi^-(\gamma)$ to the decay $K_S \rightarrow \pi^0\pi^0$ [3]. This measurement is fully inclusive of radiation for the $\pi^+\pi^-(\gamma)$ channel, allowing an unambiguous comparison with theoretical predictions. For the extraction of the phase shift $\delta_0^0 - \delta_0^2$ from $K \rightarrow \pi\pi$ we follow Ref. 2. In the presence of electromagnetic interactions, the usual isospin decomposition

^{*}<http://www.lnf.infn.it/wg/vus>

[†]The members of the FlaviaNet Kaon Working Group who contributed most to this note are V. Cirigliano (Los Alamos); and C. Gatti, M. Moulson, and M. Palutan (Frascati).

of amplitudes becomes:

$$\begin{aligned}
\mathcal{A}_{+-} &= \mathcal{A}_{1/2} + \frac{1}{\sqrt{2}}(\mathcal{A}_{3/2} + \mathcal{A}_{5/2}) \\
\mathcal{A}_{00} &= \mathcal{A}_{1/2} - \sqrt{2}(\mathcal{A}_{3/2} + \mathcal{A}_{5/2}) \\
\mathcal{A}_{+0} &= \frac{3}{2}(\mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2})
\end{aligned} \tag{1}$$

where $\mathcal{A}_{+-,+0}$ are the infrared-finite amplitudes for the decays $K^0 \rightarrow \pi^+ \pi^- (\gamma)$ and $K^+ \rightarrow \pi^+ \pi^0 (\gamma)$, respectively, and \mathcal{A}_{00} is the amplitude for the decay $K^0 \rightarrow \pi^0 \pi^0$. These amplitudes are related to the decay rate by:

$$|\mathcal{A}_n| = \left(\frac{2\sqrt{s_n}\Gamma_n}{G_n\Phi_n} \right)^{1/2} \tag{2}$$

where $n = \{+-, 00, +0\}$. The factor G_n is associated with the effect of real and virtual photons [1]. The following notation is introduced in Ref. 2:

$$\begin{aligned}
A_0 e^{i\chi_0} &= \mathcal{A}_{1/2} \\
A_2 e^{i\chi_2} &= \mathcal{A}_{3/2} + \mathcal{A}_{5/2} \\
A_2^+ e^{i\chi_2^+} &= \mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2}
\end{aligned} \tag{3}$$

where in the absence of electromagnetic interactions the A_I are the standard isospin amplitudes and the phases χ_I are identified with the s-wave $\pi\pi$ -scattering phase shifts $\delta_0^I(s = m_K^2)$. Otherwise, we have, by Eqs. (7.20) and (7.33) of Ref. 2:

$$\delta_0^0 - \delta_0^2 = \chi_0 - \chi_2 + (6.2 \pm 3.0)^\circ \tag{4}$$

and:

$$\begin{aligned}
|A_0|^2 &= a_0 g_8^2 + b_0 g_{27}^2 + c_0 g_8 g_{27} \\
|A_2|^2 &= a_2 g_8^2 + b_2 g_{27}^2 + c_2 g_8 g_{27} \\
|A_2^+|^2 &= a_2^+ g_8^2 + b_2^+ g_{27}^2 + c_2^+ g_8 g_{27}
\end{aligned} \tag{5}$$

where $g_{8,27}$ are the coefficients of the leading ($\mathcal{O}(p^2)$) octet and 27-plet weak non-leptonic chiral operators (as defined in Eqs. (2.6) and (2.7) of Ref. 2). Note that in order to arrive

at Eqs. (5), a number of higher-order chiral effective couplings are needed. Here we adopt the estimates given in Section 5 of Ref. 2.

The coefficients a , b , and c depend on the chiral renormalization scale v_χ (from the matching uncertainty in the low-energy constants) and the tree-level $\pi^0 - \eta$ mixing angle $\varepsilon^{(2)}$ given by:

$$\varepsilon^{(2)} = \frac{\sqrt{3}}{2} \frac{m_d - m_u}{2m_s - m_d - m_u}. \quad (6)$$

We have obtained the values of these coefficients from the authors of Ref. 2.

Eqs. (1), (2), and (3) can be combined to obtain

$$\begin{aligned} A_2^+ &= \frac{2}{3} |\mathcal{A}_{+0}| \\ (A_0)^2 + (A_2)^2 &= \frac{2}{3} |\mathcal{A}_{+-}|^2 + \frac{1}{3} |\mathcal{A}_{00}|^2 \\ \frac{A_2}{A_0} \cos(\chi_0 - \chi_2) &= \frac{r - 1 + (\frac{A_2}{A_0})^2 (2r - \frac{1}{2})}{\sqrt{2}(1 + 2r)} \end{aligned} \quad (7)$$

where $r = |\mathcal{A}_{+-}/\mathcal{A}_{00}|^2$. Using the expansion of Eq. (5), this system of equations can be written in terms of the three unknowns, $\chi_0 - \chi_2$, g_8 , and g_{27} . We solve this system using a numerical minimization procedure.

The experimental inputs used to obtain the amplitudes by Eq. (2) are listed in Table 1. As noted above, the BRs for $K_S \rightarrow \pi^+ \pi^- (\gamma)$ and $K_S \rightarrow \pi^0 \pi^0$ are obtained from the KLOE measurement of their ratio, which accounts for the large value of the correlation coefficient. The BR for $K^+ \rightarrow \pi^+ \pi^0$ is weakly correlated with the value of the K^+ lifetime by the fit performed in Ref. 4. These correlations are taken into account in the minimization procedure. While the KLOE measurement of the ratio of $K_S \rightarrow \pi\pi$ BRs is fully inclusive of radiation in the $\pi^+ \pi^- (\gamma)$ channel, the inclusiveness of the value for $\text{BR}(K^+ \rightarrow \pi^+ \pi^0)$ from the fit in Ref. 4 is less well defined. However, this is of lesser concern because of the dominance of the $I = 1/2$ amplitudes. With $v_\chi = 0.77$ GeV and $\varepsilon^{(2)} = 1.06 \times 10^{-2}$ we obtain:

$$\begin{aligned} g_8 &= 3.6435 \pm 0.0010 \\ g_{27} &= 0.2987 \pm 0.0006 \\ \chi_0 - \chi_2 &= (51.26 \pm 0.82)^\circ \end{aligned} \quad (8)$$

Parameter	Value	Correlation	Reference
$\text{BR}(K_S \rightarrow \pi^+ \pi^- (\gamma))$	0.69196(51)	-0.9996	[3]
$\text{BR}(K_S \rightarrow \pi^0 \pi^0)$	0.30687(51)		
τ_S	0.08958(5) ns		[5]
$\text{BR}(K^+ \rightarrow \pi^+ \pi^0)$	0.2064(8)	-0.032	[4]
τ_+	12.384(19) ns		

Table 1: Experimental inputs used to obtain g_8 , g_{27} , and $\chi_0 - \chi_2$.

The error matrix is:

$$\begin{pmatrix} 1.08 \times 10^{-6} & -7.2 \times 10^{-8} & -2.7 \times 10^{-5} \\ - & 3.8 \times 10^{-7} & 5.2 \times 10^{-5} \\ - & - & 0.68 \end{pmatrix} \quad (9)$$

We compute the systematic error by varying the chiral renormalization scale parameter v_χ from 0.5 GeV to 1 GeV, and the tree-level $\pi^0 - \eta$ mixing angle $\varepsilon^{(2)}$ from 0.6×10^{-2} to 1.5×10^{-2} , and taking half of the total variation as the uncertainty. The systematic error matrix is:

$$\begin{pmatrix} 4.0 \times 10^{-2} & 3.8 \times 10^{-4} & -0.28 \\ - & 9.3 \times 10^{-6} & -2.7 \times 10^{-3} \\ - & - & 2.0 \end{pmatrix} \quad (10)$$

Including the systematic errors and using Eq. (4) we have:

$$\begin{aligned} g_8 &= (3.644 \pm 0.001_{\text{exp}} \pm 0.200_{v_\chi \oplus \varepsilon^{(2)}}) = (3.64 \pm 0.20) \\ g_{27} &= (0.2987 \pm 0.0001_{\text{exp}} \pm 0.0030_{v_\chi \oplus \varepsilon^{(2)}}) = (0.2987 \pm 0.0030) \\ \delta_0^0 - \delta_0^2 &= (57.5 \pm 0.8_{\text{exp}} \pm 3.0_{\gamma_2} \pm 1.4_{v_\chi \oplus \varepsilon^{(2)}})^\circ = (57.5 \pm 3.4)^\circ \end{aligned} \quad (11)$$

The results in Eqs. (11) have been obtained using the central values for the higher-order chiral couplings estimated in Ref. 2 within the large- N_C expansion. The quoted uncertainty reflects only the uncertainty in the matching scale. As already pointed out in Ref. 2 (Section 7.3), the extraction of g_8 and g_{27} is rather sensitive to the input on the chiral couplings. For instance, even a simple variant of the large- N_C procedure leads

to changes in g_8 and g_{27} at the 10% level [2]. This implies that the values for $g_{8,27}$ quoted in Eq. (11) are affected by an unknown systematic offset of at least 10%. (See also the analysis of Ref. 6.) On the other hand, the extraction of the phase difference $\chi_0 - \chi_2$ is quite insensitive to the input on the effective chiral couplings. The result for the phase difference $\delta_0^0 - \delta_0^2$ instead depends quite sensitively on the estimate of the isospin-breaking correction of Eq. (4).

It is interesting to compare the present result for $\delta_0^0 - \delta_0^2$ with predictions from phenomenological evaluations. The Roy equations [7] determine the $\pi\pi$ scattering amplitude in terms of its imaginary part at intermediate energies, up to two subtraction constants: the s-wave scattering lengths a_0^0 and a_0^2 . Colangelo, Gasser, and Leutwyler [8] obtain values for a_0^0 and a_0^2 by matching a representation of the $\pi\pi$ scattering amplitude from $\mathcal{O}(p^6)$ calculations in chiral perturbation theory with a phenomenological representation based on the Roy equations. They obtain $\delta_0^0 - \delta_0^2 = (47.7 \pm 1.5)^\circ$, which differs from our result by 2.6σ . Kamiński, Peláez, and Ynduráin [9] fit experimental $\pi\pi$ scattering amplitudes at both low and high energies with parameterizations that satisfy analyticity at low energy, constrained to satisfy the forward dispersion relations and the Roy equations. They obtain $\delta_0^0 - \delta_0^2 = (50.9 \pm 1.2)^\circ$, which differs from our result by 1.8σ . It is important to note that their input data includes low-energy s-wave phase shift determinations from K_{e4} and $K \rightarrow 2\pi$ decays, including a preliminary result for $\delta_0^0 - \delta_0^2$ from this update, differing very little from the value presented here. The significant discrepancies between our result, based on K_S and K^+ BR measurements, and the results of phenomenological analyses of $\pi\pi$ scattering (with or without constraints from chiral symmetry) are puzzling and deserve further investigation.

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